Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

■ : explanation
∵ is 'because'
∴ is 'therefore'

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WMA11/01**

Mathematics

October 2021

International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. Find

$$\int 12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4}\,dx$$

giving each term in simplest form.

(5)

integration

① Write in form easier for integration.

$$12x^3 + \frac{1}{6\sqrt{x}} - \frac{3}{2x^4} = 12x^3 + \left(\frac{1}{6} \times \frac{1}{\sqrt{x}}\right) - \left(\frac{3}{2} \times \frac{1}{x^4}\right)$$

$$= 12x^3 + \left(\frac{1}{6} \times \frac{1}{x^{1/2}}\right) - \left(\frac{3}{2} \times \frac{1}{x^4}\right)$$

① indicies rule: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$$= 12x^3 + \frac{1}{6}x^{-1/2} - \frac{3}{2}x^{-4}$$

② indicies rule: $\frac{a}{x^b} = ax^{-b}$

② Integrate.

$$\int 12x^3 + \frac{1}{6}x^{-1/2} - \frac{3}{2}x^{-4}\,dx$$

$$= \left[\left(\frac{12}{3+1}x^{3+1}\right) + \left(\frac{1/6}{-1/2+1}x^{-1/2+1}\right) + \left(\frac{-3/2}{-4+1}x^{-4+1}\right)\right]$$

$$= \frac{12}{4}x^4 + \frac{1}{3}x^{1/2} + \frac{1}{2}x^{-3} + C$$

↳ DON'T FORGET !!! or will lose a mark.

③ Simplify

$$\therefore \quad 3x^4 + \frac{1}{3}x^{1/2} + \frac{1}{2}x^{-3} + C$$

Leave blank

Question 1 continued

Q1

(Total 5 marks)

P 6 6 6 4 5 A 0 3 3 2

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = 3x^5 + 4x^3 - x + 5$$

The points P and Q lie on the curve.

The gradient of the curve at both point P and point Q is 2

Find the x coordinates of P and Q.

(5)

$m = 2$

As both P & Q lie on curve, the x-coordinates of P & Q will reveal the gradient when put in the gradient function (differential) dy/dx. ∴ we will first find dy/dx then solve for x when $dy/dx = 2$ to find x-coordinate of P & Q.

① Differentiate $y = 3x^5 + 4x^3 - x^1 + 5x^0$ $\because x^0 = 1$

$dy/dx = 5(3x^{5-1}) + 3(4x^{3-1}) + 1(-x^{1-1}) + 0(5x^{0-1})$

$= 15x^4 + 12x^2 - 1$

② equate dy/dx to 2.

$dy/dx\big|_{x=?} = 15x^4 + 12x^2 - 1 = 2$

③ Solve for x.

$15x^4 + 12x^2 - 1 = 2$

-2 $15x^4 + 12x^2 - 3 = 0$ -2

let $y = x^2 \Rightarrow x^4 = x^{2\times 2} = (x^2)^2 = y^2$

$\therefore 15y^2 + 12y - 3 = 0$

Question 2 continued

FACTORISE : $3(5y^2+4y-1)=0$

$3(5y-1)(y+1)=0$

↓ substitute x^2 back

$3(5x^2-1)(x^2+1)=0$

Solve for x : $3(5x^2-1)(x^2+1)=0$

$5x^2-1=0$

+1 $5x^2=1$ +1

÷5 $x^2=\frac{1}{5}$ ÷5

square root $x=\pm\sqrt{\frac{1}{5}}$

$\therefore x=\pm\frac{1}{\sqrt{5}}$ ($\sqrt{1}=1$)

$x^2+1=0$

−1 $x^2=-1$ −1

square root UNDEFINED square root

$\because$ can't have square root of a negative number

$\therefore$ x-value of ? ∉ Q is one of $\pm\frac{1}{\sqrt{5}}$

Q2

(Total 5 marks)

Turn over

P 6 6 6 4 5 A 0 5 3 2

Leave blank

3. (i) Solve

$$\frac{3}{x} > 4$$

(3)

(ii)

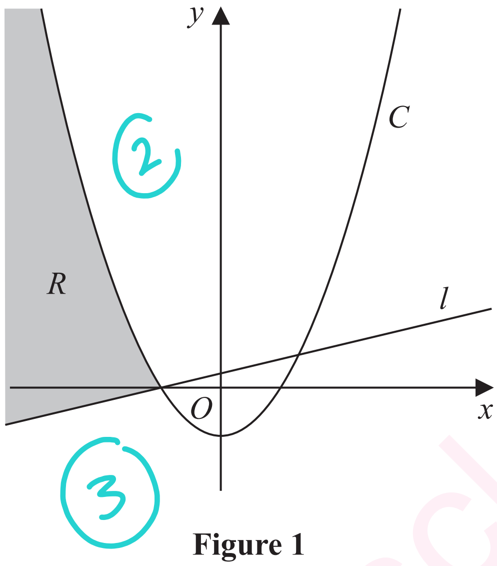


Figure 1

Figure 1 shows a sketch of the curve C and the straight line l.

The infinite region R, shown shaded in Figure 1, lies in quadrants 2 and 3 and is bounded by C and l only.

Given that

- l has a gradient of 3
- C has equation $y = 2x^2 - 50$
- C and l intersect on the negative x-axis

use inequalities to define the region R.

(3)

i)

$$\frac{3}{x} > 4 \quad \times x^2$$

$$3x > 4x^2 \quad -3x$$

$$0 > 4x^2 - 3x$$

$$0 > x(4x-3)$$

Consider $x = 0$ & $4x - 3 = 0$

$x_1 = 0 \qquad x_2 = \frac{3}{4}$

$\therefore \quad 0 < x < \frac{3}{4}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

ii) $\boxed{1}$ find equation l.

① l has gradient 3 & intersects C. ~~Intersectio~~ C at negative x-axis meaning at $y=0$. Find coordinates by substituting $y=0$ in C: $y = 2x^2 - 50$.

$$\begin{aligned} 0 &= 2x^2 - 50 \\ +50 \quad 50 &= 2x^2 \quad +50 \\ \div 2 \quad 25 &= x^2 \quad \div 2 \\ \text{square root} \quad \pm 5 &= x \quad \text{square root} \end{aligned}$$

$\therefore x = -5 \Rightarrow (-5, 0)$

$\because$ negative x-axis

② find equation of l using line passing through (a, b) and gradient m

equation: $(y - b) = m(x - a)$

$a = -5$
$b = 0$
$m = 3$

$$(y - 0) = 3(x - (-5))$$

$$y = 3(x + 5)$$

$$\therefore y = 3x + 15$$

$\boxed{2}$ Find inequalities

① First inequality is $x \leq -5$

$\hookrightarrow \leq \because$ solid lines & NOT dashed (solid / dashed)

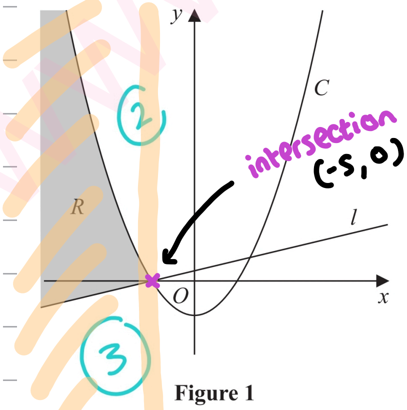


Figure 1

BUT markscheme allows for use of '<', although '≤' is more accurate. Only use 1 symbol for all the inequalities (either < or ≤, don't use both)

(Total 6 marks)

Q3

P 6 6 6 4 5 A 0 7 3 2

② Second inequality:

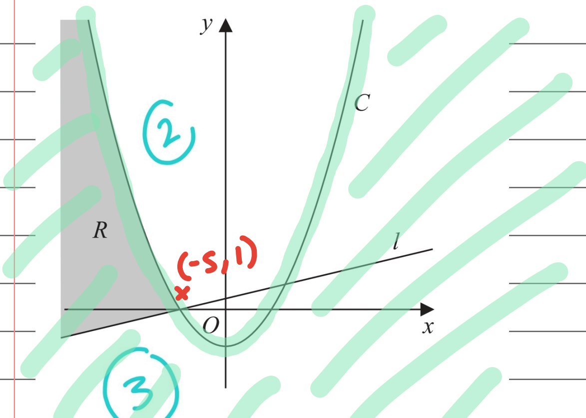


Figure 1

Curve $y = 2x^2 - 50$

To find inequality, choose point OUTSIDE valid region, and make it FALSE.

$1 = 2(-5)^2 - 50$

$1 = 0$

MAKE IT FALSE $\Rightarrow$ $1 \leq 0$

$\therefore \; y \leq 2x^2 - 50$

③ third inequality

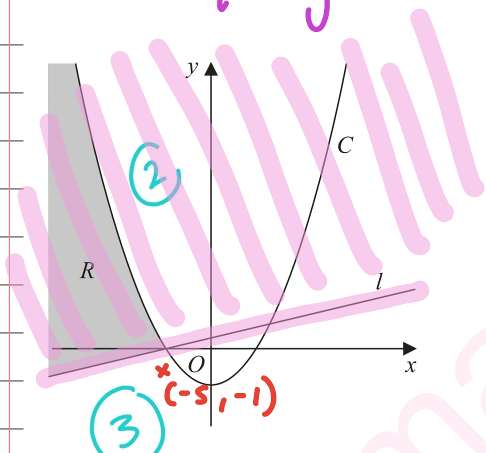


Figure 1

Line $y = 3x + 15$

To find inequality, choose point OUTSIDE valid region, and make it FALSE.

$-1 = 3(-5) + 15$

$-1 = 0$

MAKE IT FALSE $\Rightarrow$ $-1 \geq 0$

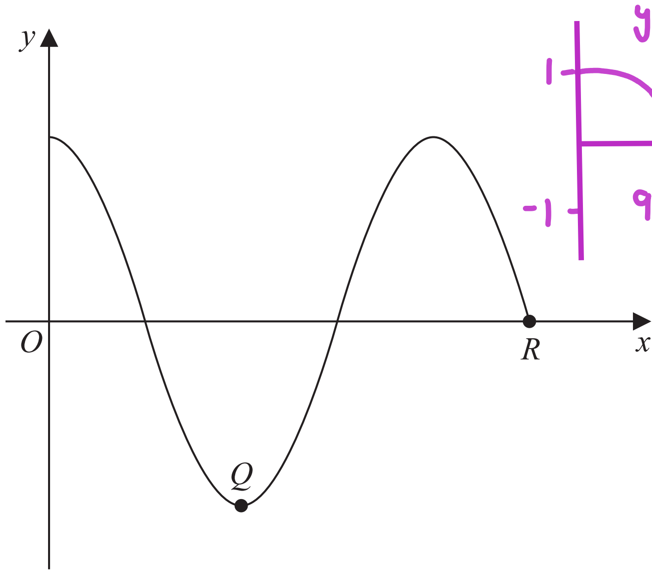
$\therefore \; y \geq 3x + 15$

ANSWER:

$x \leq -5 \;, \quad y \leq 2x^2 - 50 \;, \quad y \geq 3x + 15$

Leave blank

4.



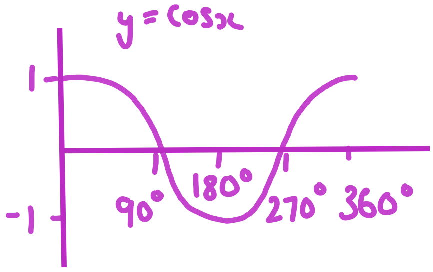


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \cos 2x^{\circ} \qquad 0 \leqslant x \leqslant k$$

← UNITS

The point Q and the point $R(k, 0)$ lie on the curve and are shown in Figure 2.

(a) State

(i) the coordinates of Q,

(ii) the value of k.

(3)

(b) Given that there are exactly two solutions to the equation

$$\cos 2x^{\circ} = p \qquad \text{in the region } 0 \leqslant x \leqslant k$$

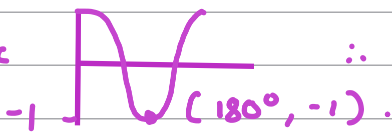
find the range of possible values for p.

(2)

a) $y = \cos 2x$ same as $y = f(2x)$ ∴ horizontal squash by factor 2.
So we divide x-coordinates of $y = \cos x$ by 2, and
y-coordinates are unchanged.

i) Q is minimum.

in $y = \cos x$



∴ in $y = \cos 2x \Rightarrow \left(\frac{180}{2}, -1\right)$

∴ $Q(90°, -1)$

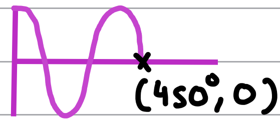
Leave blank

Question 4 continued

ii) $0 \le x \le k$

Figure 2 touches x-axis 3 times.

in $y = \cos x$ this is:



$0 \le x \le 450$

$\therefore$ for $y = 2\cos x$ (Figure 2) $\Rightarrow$ $x \le \frac{450}{2}$

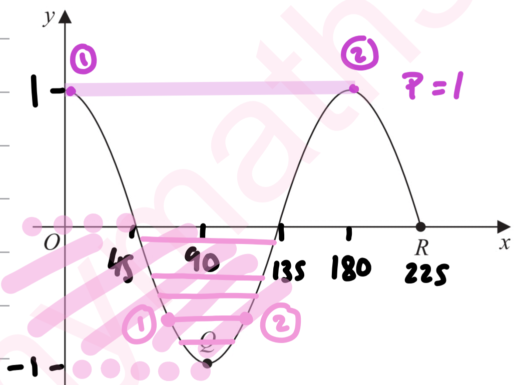
$\therefore$ $0 \le x \le 225$

$\therefore$ $k = 225°$

b) In region $0 \le x \le k$, exactly 2 solutions for $\cos 2x = p$

region is $0 \le x \le 225$ from part a) ii)

P can be :



range is $-1 < p < 0$

NOT $\le$ $\because$ at -1 and 0, P would intersect $y = \cos 2x$ $\underline{\underline{3}}$ times.

$\therefore$ $p = 1$ & $-1 < p < 0$

Q4

(Total 5 marks)

P 6 6 6 4 5 A 0 9 3 2

5. The line l_1 has equation $3y - 2x = 30$

The line l_2 passes through the point $A(24, 0)$ and is perpendicular to l_1

Lines l_1 and l_2 meet at the point P

(a) Find, using algebra and showing your working, the coordinates of P.

(5)

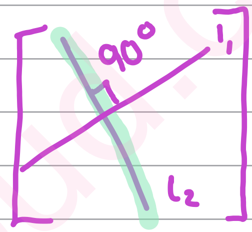
Given that l_1 meets the x-axis at the point B,

(b) find the area of triangle BPA.

(3)

a) 1 find equation of l_2.

normal is Perpendicular to l_2



$\therefore$ we find gradient of normal (M_n) using Perpendicular gradient rule $M_{normal} \times M_{curve} = -1$

gradient of l_1 will be found by rearranging equation in form $y = mx + c$

$$l_1: \quad 3y - 2x = 30$$
$$+2x \qquad 3y = 2x + 30 \qquad +2x$$
$$\div 3 \qquad y = \frac{2}{3}x + 10 \qquad \div 3$$

Perpendicular gradient rule:

$$M_n \times \frac{2}{3} = -1 \qquad \div 2/3$$
$$M_n = -3/2$$

Find equation of l_2 using line passing through (a, b) and gradient M

equation: $(y - b) = M(x - a)$

$a = 24$

$b = 0$

$M = -3/2$

$$(y - 0) = -3/2(x - 24)$$

$$y = -3/2(x - 24) \implies y = -3/2x + 36$$

P 6 6 6 4 5 A 0 1 0 3 2

Question 5 continued

② equate l_1 & l_2 together to find point of intersection P.

$$l_1 : y = \tfrac{2}{3}x + 10$$
$$l_2 : y = -\tfrac{3}{2}x + 36$$

$$\tfrac{2}{3}x + 10 = -\tfrac{3}{2}x + 36$$

$+\tfrac{3}{2}x$ $\qquad \tfrac{13}{6}x + 10 = 36 \qquad$ $+\tfrac{3}{2}x$

-10 $\qquad \tfrac{13}{6}x = 26 \qquad$ -10

$\div \tfrac{13}{6}$ $\qquad x = 12 \qquad$ $\div \tfrac{13}{6}$

② Substitute x-value into equation of either l_1 or l_2 to find y-coordinate.

$$l_1 : y = \tfrac{2}{3}x + 10$$
$$y = \tfrac{2}{3}(12) + 10 = 8 + 10 = 18$$
$$\therefore y = 18$$

$\therefore$ P(12, 18)

b) A(24, 0) P(12, 18)

B is when l_1 intersects x-axis $\therefore y = 0$.

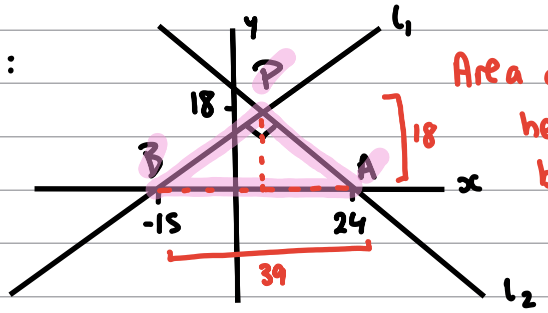
$$l_1 : y = \tfrac{2}{3}x + 10$$
$$0 = \tfrac{2}{3}x + 10$$

-10 $\qquad -10 = \tfrac{2}{3}x \qquad$ -10

$\div \tfrac{2}{3}$ $\qquad -15 = x \qquad$ $\div \tfrac{2}{3}$

B(−15, 0)

Triangle BPA :



Area of a triangle: $A = \tfrac{1}{2}bh$

height = 18

base = 24 − (−15) = 39

$\therefore A = \tfrac{1}{2} \times 18 \times 39 =$

$\therefore$ Area BPA = 351 units^2

Turn over

P66645A01132

Leave blank

Question 5 continued

Leave blank

Question 5 continued

Q5

(Total 8 marks)

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 2(x+1)(x-3)^2$$

(a) Sketch a graph of C.

Show on your graph the coordinates of the points where C cuts or meets the coordinate axes.

(3)

(b) Write $\mathrm{f}(x)$ in the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are constants to be found.

(3)

(c) Hence, find the equation of the tangent to C at the point where $x = \frac{1}{3}$

(4)

Working out:

a) when $f(x) = 0$

$x+1=0 \Rightarrow x=-1$

$x-3=0 \Rightarrow x=3$

when $x = 0$

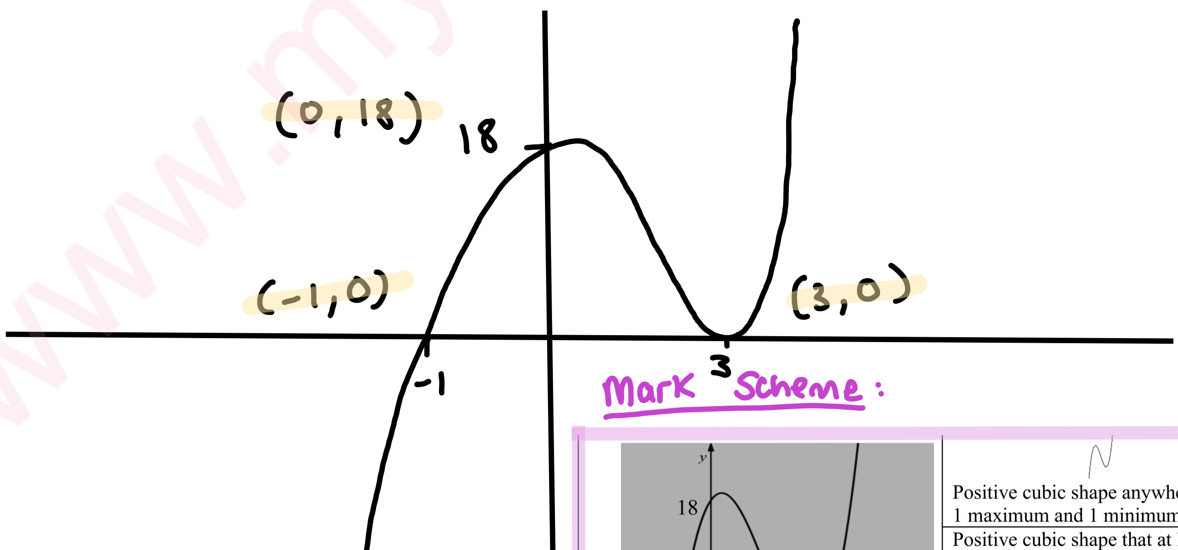
$f(0) = 2(0+1)(0-3)^2$

$y = 18$

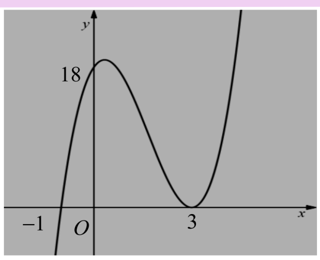
* graph is cubic $\because$ when brackets are expanded, there will be x^3 value. Also positive cubic as it is $+x^3$ $\therefore$ Shape is N

* Passes through -1 and touches -3 at x-axis $\because (x-3)^2$

Graph:



Mark Scheme:

	Positive cubic shape anywhere with 1 maximum and 1 minimum
	Positive cubic shape that at least reaches the x-axis at $x = -1$ and with a minimum on the x-axis at $x = 3$
	y intercept at 18. Must correspond with their sketch
For the intercepts allow as numbers as above or allow as coordinates e.g. (18, 0), (0, –1), (0, 3) as long as they are marked in the correct place.	

Question 6 continued

b) expanding brackets

$f(x) = 2(x+1)(x-3)^2 = 2(x+1)(x-3)(x-3) = 2(x+1)(x^2-3x-3x+9)$

$= 2(x+1)(x^2-6x+9) = 2(x^3-6x^2+9x+x^2-6x+9)$

$= 2(x^3-5x^2+3x+9) = 2x^3-10x^2+6x+18$

$\therefore f(x) = 2x^3-10x^2+6x+18$ $\quad a=2 \quad b=-10 \quad c=6 \quad d=18$

c) tangent means gradient of tangent is same as gradient of equation

① to find gradient of tangent, substitute $x=\frac{1}{3}$ into gradient function (differential, $f'(x)$) of $f(x)$.

$\because x^0 = 1$

$f(x) = 2x^3 - 10x^2 + 6x^1 + 18x^0$

$f'(x) = 3(2x^{3-1}) + 2(-10x^{2-1}) + 1(6x^{1-1}) + 0(18x^{0-1})$ from part (b)

$= 6x^2 - 20x + 6$

$f'(\frac{1}{3}) = 6(\frac{1}{3})^2 - 20(\frac{1}{3}) + 6 = 0$

$\therefore$ gradient $= 0$

② find y-coordinate of tangent by substituting $x=\frac{1}{3}$ into $f(x)$.

$f(\frac{1}{3}) = 2(\frac{1}{3})^3 - 10(\frac{1}{3})^2 + 6(\frac{1}{3}) + 18 = \frac{512}{27}$

③ find equation of tangent using line passing through (a, b) and gradient m

equation: $(y-b) = m(x-a)$

$a = \frac{1}{3}$

$b = \frac{512}{27}$

$m = 0$

$(y - \frac{512}{27}) = 0(x - \frac{1}{3})$

$y - \frac{512}{27} = 0$

$\therefore$ equation of tangent is $y = \frac{512}{27}$

Question 6 continued

P 6 6 6 4 5 A 0 1 6 3 2

Question 6 continued

(Total 10 marks)

Q6

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7.

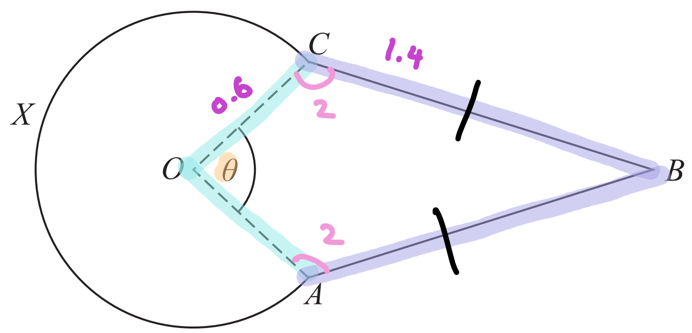


Figure 3

Figure 3 shows the design for a sign at a bird sanctuary.

The design consists of a kite *OABC* joined to a sector *OCXA* of a circle centre *O*.

In the design

- $OA = OC = 0.6\,\text{m}$ = radius
- $AB = CB = 1.4\,\text{m}$
- Angle OAB = Angle OCB = 2 radians → UNITS!!
- Angle $AOC = \theta$ radians, as shown in Figure 3

Making your method clear,

(a) show that $\theta = 1.64$ radians to 3 significant figures,

(4)

(b) find the perimeter of the sign, in metres to 2 significant figures,

(2)

(c) find the area of the sign, in m^2 to 2 significant figures.

(4)

Data booklet

Pure Mathematics P1

Mensuration

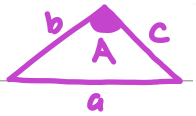
Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

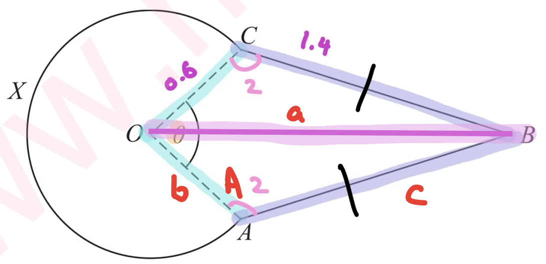
→ **Cosine rule**

$a^2 = b^2 + c^2 - 2bc\cos A$

a) ① Using Cosine rule where



$a^2 = b^2 + c^2 - 2bc\cos A$



find length OB.

$OB^2 = 0.6^2 + 1.4^2 - 2(0.6)(1.4)\cos 2 = 0.36 + 1.96 + 1.68\cos 2$

$= 2.32 - (-0.699126\ldots) = 3.01912\ldots$

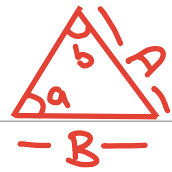
$OB^2 = 3.019126\ldots$

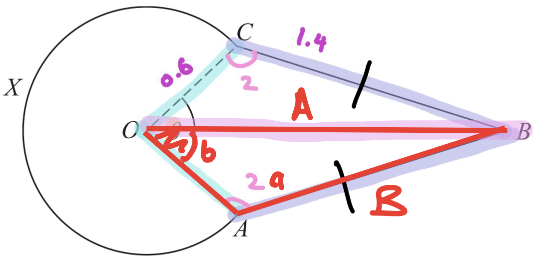
$\therefore OB = 1.737563\ldots \approx 1.7376$

P 6 6 6 4 5 A 0 1 8 3 2

Question 7 continued

(2) Use Sine rule where $\frac{\sin a}{A} = \frac{\sin b}{B}$





Find <AOB.

Angle AOC will be 2 x <AOB

$$\frac{\sin 2}{1.7376} = \frac{\sin \angle AOB}{1.4}$$

×1.4 ×1.4

$0.732629... = \sin \angle AOB$

inverse sine inverse sine

$\sin^{-1}(0.732629...) = \angle AOB$

$\therefore \angle AOB = 0.8221767... \approx 0.82218$

(3) $2 \times \angle AOB = \angle AOC$

$2 \times 0.82218 = 1.64436 \approx 1.64$ (3 s.f.)

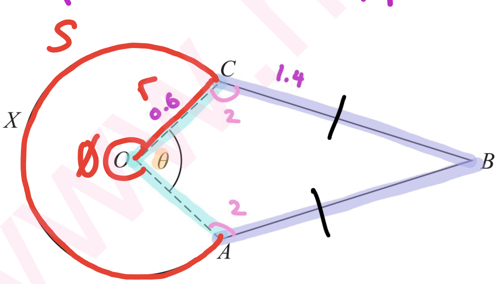
$\therefore \angle AOC = 1.64$ rad (3 s.f.)

b) Perimeter of sign is AB + BC + AXC

$AB + BC + AXC = 1.4 + 1.4 + AXC = 2.8 + AXC$

to find Arc AXC, formula $S = r\theta$

Angle in a circle is 2π rad (360°)



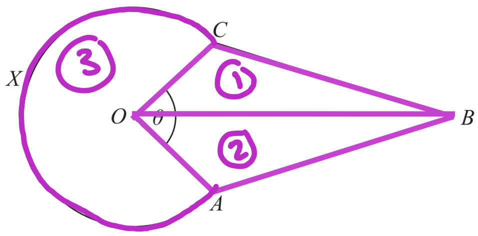
$S = 0.6 \times (2\pi - 1.64) = 2.785911...$
≈ 2.7859

Perimeter $= 2.8 + AXC = 2.8 + 2.7859 = 5.5859$

$\therefore$ Perimeter $= 5.6$ m (2 s.f.)

Question 7 continued

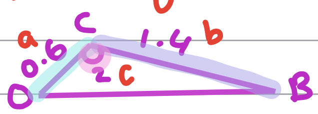
c) Area of Sign. Split sign into 3 shapes:



$\triangle OCB = \triangle OAB$

$\therefore$ Area of ① & ② are equal.

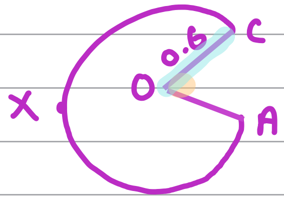
Area of a triangle: $A = \frac{1}{2} ab \sin C$

① : 

$A = \frac{1}{2}(0.6)(1.4) \sin 2 = 0.42 \sin 2$
$= 0.3819049\ldots$

① + ② = 2 × ① = $2 \times 0.3819049\ldots = 0.763809\ldots$
$\approx \underline{0.7638}$

Sector Area: $A = \frac{1}{2} r^2 \theta$



$A = \frac{1}{2} \times 0.6^2 \times (2\pi - 1.64)$
$= 0.835773\ldots$
$\approx \underline{0.8358}$

Total Area = ① + ② + ③ = 2① + ③
$= 0.7638 + 0.8358 = 1.5996$

$\therefore$ Area $= 1.6\text{ m}^2$ (2 s.f.)

Leave blank

Question 7 continued

Q7

(Total 10 marks)

P 6 6 6 4 5 A 0 2 1 3 2

8.

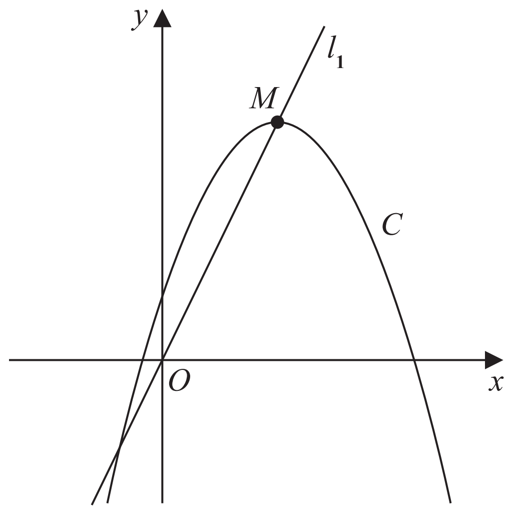


Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 4 + 12x - 3x^2$$

The point M is the maximum turning point on C.

(a) (i) Write $4 + 12x - 3x^2$ in the form

$$a + b(x + c)^2$$

where a, b and c are constants to be found.

(ii) Hence, or otherwise, state the coordinates of M.

(5)

The line l_1 passes through O and M, as shown in Figure 4.

A line l_2 touches C and is parallel to l_1

(b) Find an equation for l_2

(5)

a) i) Completing the Square :

if $y = x^2 + bx + c$

$y = \left(x + \frac{b}{2}\right)^2 + c - \left(\frac{b}{2}\right)^2$

$y = -3x^2 + 12x + 4$

$y = -3\left(x^2 - 4x - \frac{4}{3}\right)$

$y = -3\left(\left(x + \frac{-4}{2}\right)^2 + \left(-\frac{4}{3}\right) - \left(\frac{-4}{2}\right)^2\right) = -3\left((x-2)^2 - \frac{16}{3}\right)$

$= -3(x-2)^2 + 16$

$\therefore y = 16 - 3(x-2)^2$ $\quad a = 16 \quad b = -3 \quad c = -2$

P 6 6 6 4 5 A 0 2 2 3 2

Question 8 continued

ii) to find coordinates of maximum point:

$$y = \left(x + \tfrac{b}{2}\right)^2 + \underbrace{c - \left(\tfrac{b}{2}\right)^2}_{y\text{-coordinate}}$$

inverse is x-coordinate

Explanation: when $y = c - (b/2)^2$ $\quad c - (b/2)^2 = (x + b/2)^2 + c - (b/2)^2$

$0 = (x + b/2)^2$

$\therefore x = -b/2$

when $x = -b/2$ $\quad y = (-b/2 + b/2)^2 + c - (b/2)^2$

$\therefore y = c - (b/2)^2$

$\therefore \quad -3(x-2)^2 + 16$

$\to -x \qquad \to y$

$\therefore M(2, 16)$

b) $l_2 \parallel l_1$ (parallel) $\therefore$ both have same gradient

① find gradient of l_1, which will give us gradient of l_2.

gradient formula $m = \dfrac{y_1 - y_2}{x_1 - x_2}$

l_1 passes through $O(\underset{x_2}{0}, \underset{y_2}{0})$ & $M(\underset{x_1}{2}, \underset{y_1}{16})$

$$m_{l_2} = m_{l_1} = \frac{16-0}{2-0} = \frac{16}{2} = 8$$

② l_2 equation is $y = 8x + c$ where c is a constant to be found.

l_2 'touches' C, so 1 real root. Use discriminant: $b^2 - 4ac = 0$ for 1 real root, to find the constant.

* equate l_2 with C to form equation.

$l_2 \to 8x + c = 4 + 12x - 3x^2 \leftarrow C$

$-8x \quad c = 4 + 4x - 3x^2 \quad -8x$

$-c \quad 0 = 4 - c + 4x - 3x^2 \quad -c$

$\therefore -3x^2 + 4x + (4 - c) = 0$

P 6 6 6 4 5 A 0 2 3 3 2

Question 8 continued

a b c

$$-3x^2 + 4x + (4-c) = 0$$

$$b^2 - 4ac = 0$$

$$(4)^2 - 4(-3)(4-c) = 0$$

$$16 + 12(4-c) = 0 \rightarrow 16 + 48 - 12c = 0$$

$$64 - 12c = 0$$

$$-64 \quad -12c = -64 \quad -64$$

$$\div -12 \quad c = \frac{-64}{-12} \quad \div -12$$

$$\therefore \quad c = \frac{16}{3}$$

$y = mx + c$

$m = 8$

$c = \frac{16}{3}$

$$\therefore \quad y = 8x + \frac{16}{3}$$

Leave blank

Question 8 continued

Q8

(Total 10 marks)

P 6 6 6 4 5 A 0 2 5 3 2

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9. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

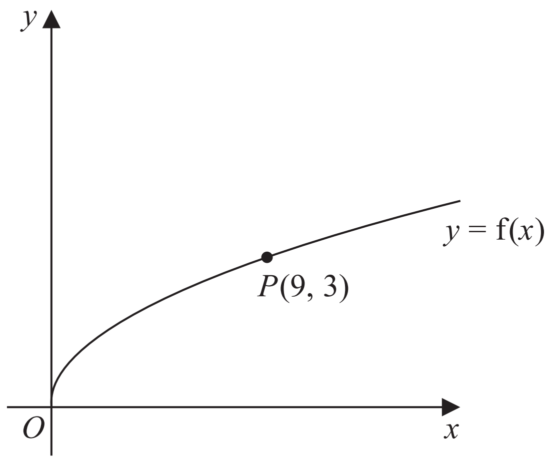


Figure 5

Figure 5 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \sqrt{x} \qquad x > 0$$

The point $P(9, 3)$ lies on the curve and is shown in Figure 5.

On the next page there is a copy of Figure 5 called Diagram 1.

(a) On Diagram 1, sketch and clearly label the graphs of

$$y = \mathrm{f}(2x) \quad \text{and} \quad y = \mathrm{f}(x) + 3$$

Show on each graph the coordinates of the point to which P is transformed.

(3)

The graph of $y = \mathrm{f}(2x)$ meets the graph of $y = \mathrm{f}(x) + 3$ at the point Q.

(b) Show that the x coordinate of Q is the solution of

$$\sqrt{x} = 3\left(\sqrt{2} + 1\right)$$

(3)

(c) Hence find, in simplest form, the coordinates of Q.

(3)

a) Working out f(2x): $f(x) = \sqrt{x}$

$f(2x) = \sqrt{2x}$ horizontal squash by 2.

inside f(x) bracket so only x-coordinates affected & we do the inverse: ÷2.

$O(0,0) \rightarrow (\frac{0}{2}, 0) \rightarrow (0,0)$

$P(9,3) \rightarrow (\frac{9}{2}, 3) \rightarrow (4.5, 3)$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

P 6 6 6 4 5 A 0 2 6 3 2

Question 9 continued

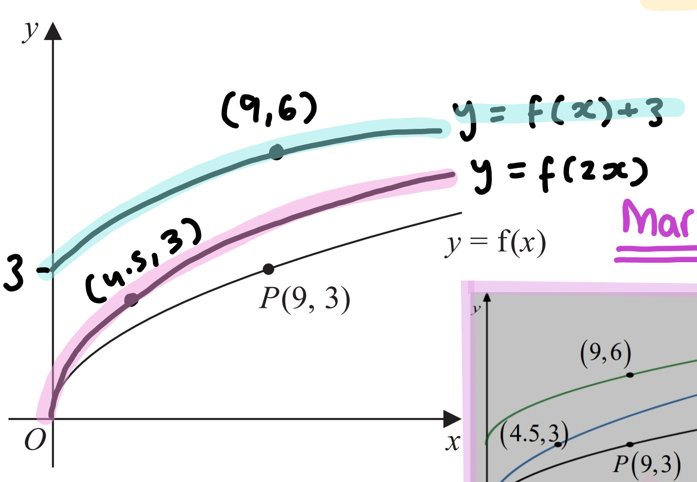
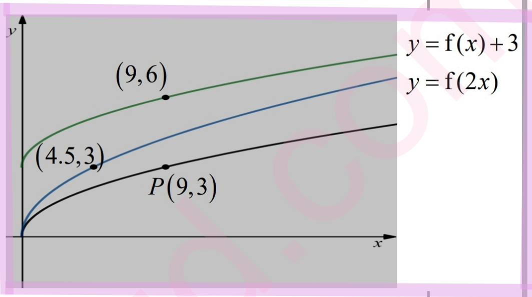


Diagram 1

Mark scheme:



Turn over for a copy of Diagram 1 if you need to redraw your graphs.

working out $f(x)+3$: $f(x)=\sqrt{x}$

$f(x)+3=\sqrt{x}+3$ ← translation $\begin{pmatrix}0\\3\end{pmatrix}$, 3 units up.

outside f(x) bracket ∴ only y-coordinates affected

$O(0,0) \rightarrow (0,0+3) \rightarrow (0,3)$

$P(9,3) \rightarrow (9,3+3) \rightarrow (9,6)$

b) equate $f(2x)$ & $f(x)+3$ to find intersection Q.

$f(2x)=f(x)+3$

$\sqrt{2x}=\sqrt{x}+3$

$-\sqrt{x}$ $\quad \sqrt{2x}-\sqrt{x}=3$ $\quad -\sqrt{x}$

$\sqrt{2}\sqrt{x}-\sqrt{x}=3$

$\sqrt{x}(\sqrt{2}-1)=3$

$\div(\sqrt{2}-1)$ $\quad \sqrt{x}=\frac{3}{(\sqrt{2}-1)}$ $\quad \div(\sqrt{2}-1)$

$\therefore \sqrt{x}=\frac{3}{(\sqrt{2}-1)}$

rationalising surds:

$\sqrt{x}=\frac{3}{(\sqrt{2}-1)}\times\frac{(\sqrt{2}+1)}{(\sqrt{2}+1)}=\frac{3(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}=\frac{3(\sqrt{2}+1)}{2+\sqrt{2}-\sqrt{2}-1}$

$=\frac{3(\sqrt{2}+1)}{2-1}=\frac{3(\sqrt{2}+1)}{1}=3(\sqrt{2}+1)$

Question 9 continued

$\therefore$ x-coordinate of Q is solution of:

$$\sqrt{x} = 3(\sqrt{2}+1)$$

c) ① to find x-coordinate of Q, solve $\sqrt{x} = 3(\sqrt{2}+1)$ from part (b).

$$\sqrt{x} = 3(\sqrt{2}+1)$$
$$\sqrt{x} = 3\sqrt{2}+3$$
square $\quad x = (3\sqrt{2}+3)^2 \quad$ square

$x = (3\sqrt{2}+3)(3\sqrt{2}+3) = 18 + 9\sqrt{2} + 9\sqrt{2} + 9 = 27 + 18\sqrt{2}$

in mark scheme also written as $9(3+2\sqrt{2})$

$\therefore\ x = 27 + 18\sqrt{2}$

② find y-coordinate of Q by substituting x into either $f(2x)$ or $f(x)+3$.

$f(x+3) = \sqrt{x}+3$

$f((27+18\sqrt{2})+3) = (\sqrt{27+18\sqrt{2}}) + 3 = (\sqrt{(3\sqrt{2}+3)^2}) + 3$

$\because$ $27+18\sqrt{2}$ is $(3\sqrt{2}+3)^2$ from finding x.

$= (3\sqrt{2}+3) + 3 = 3\sqrt{2}+6$

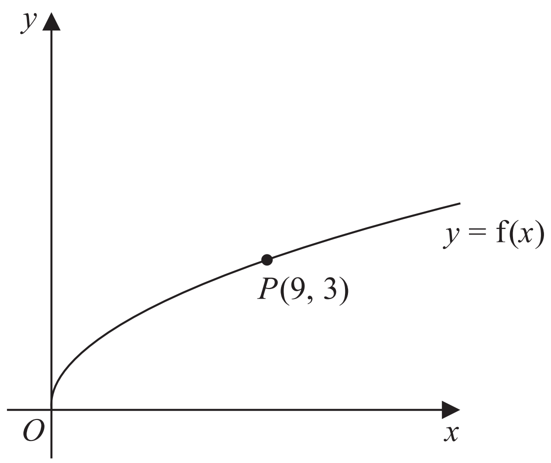
$\therefore\ y = 3\sqrt{2}+6$

$\therefore\ Q((27+18\sqrt{3}), (3\sqrt{2}+6))$

Leave blank

Question 9 continued

Only use this copy if you need to redraw your graphs.



Copy of Diagram 1

(Total 9 marks)

Q9

10. A curve has equation $y = f(x)$, $x > 0$

Given that

- $f'(x) = ax - 12x^{\frac{1}{3}}$, where a is a constant
- $f''(x) = 0$ when $x = 27$
- the curve passes through the point $(1, -8)$

(a) find the value of a.

(3)

(b) Hence find $f(x)$.

(4)

$$f(x) \underset{\text{integrate (b)}}{\overset{\text{differentiate}}{\rightleftarrows}} f'(x) \underset{\text{integrate}}{\overset{\text{differentiate (a)}}{\rightleftarrows}} f''(x)$$

a) ① find equation of $f''(x)$ by differentiating $f'(x)$.

$$f'(x) = ax^1 - 12x^{\frac{1}{3}}$$

$$f''(x) = 1(ax^{1-1}) + \frac{1}{3}(-12x^{\frac{1}{3}-1}) = a - 4x^{-2/3}$$

② When $x = 27$, $f''(x) = 0$

$$f''(27) = 0$$

$$f''(27) = a - 4(27)^{-2/3} = 0$$

$$= a - \frac{4}{9} = 0$$

$$+\frac{4}{9} \quad a = \frac{4}{9} \quad +\frac{4}{9}$$

$$\therefore \quad a = \frac{4}{9}$$

b) find $f(x)$ by integrating $f'(x)$.

① integrate

part (a)

$$f(x) = \int f'(x)\,dx = \int ax - 12x^{\frac{1}{3}}\,dx = \int \frac{4}{9}x^1 - 12x^{\frac{1}{3}}\,dx$$

$$= \left[\left(\frac{\frac{4}{9}}{1+1}x^{1+1}\right) + \left(\frac{-12}{\frac{1}{3}+1}x^{\frac{1}{3}+1}\right)\right] = \frac{2}{9}x^2 - 9x^{\frac{4}{3}} + C$$

② Find C. Curve passes through $(1, -8)$ so substitute into $f(x)$.

$$f(1) = \frac{2}{9}(1)^2 - 9(1)^{\frac{4}{3}} + C = -8$$

$$\frac{2}{9} - 9 + C = -8$$

Question 10 continued

$$-\frac{79}{9} + C = -8$$

$$+\frac{79}{9} \qquad C = \frac{7}{9} \qquad +\frac{79}{9}$$

$$\therefore \boxed{f(x) = \frac{2}{9}x^2 - 9x^{\frac{4}{3}} + \frac{7}{9}}$$

Leave blank

Question 10 continued

Q10

(Total 7 marks)

TOTAL FOR PAPER IS 75 MARKS

END